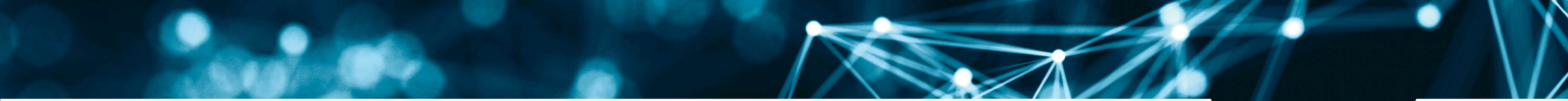

Rotating thermal convection under generalised quasilinear approximations

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We model rotating Rayleigh–Bénard convection in 3D direct numerical simulations of a periodic box with Boussinesq perturbations, subject to inclined global rotation. Vertical boundary temperatures are fixed locally, with a linear meridional gradient (T_y). We vary T_y and convective driving. Emergent flows can include thermal winds, anisotropic turbulence, and giant vortices. Fully nonlinear (NL) runs provide benchmarks for testing quasilinear (QL) and general quasilinear (GQL) approximate dynamics. In horizontal planes, each variable (q) splits into a Fourier "low" background (q_L) at $|k_x|,|k_y|<\Lambda$, and a "high" variable for fluctuations (q_H). Varying the GQL cutoff (Λ=∞,10,5,1,0) selectively impairs spectral power transfers, affecting flow morphologies, entropy, variability, local and global statistics. Versions of GQL usually improve upon the accuracy of QL (Λ=0), but low-Λ can produce spurious inverse cascades. Higher Λ improves realism more readily for models with stronger thermal winds. GQL succeeds generally even in peculiar cases that never settle to one unimodal steady state. Accuracy depends on the directness of boundary conditions, simple vertical profiles, and raising Λ to enclose the most active Fourier modes. However, spatial anisotropies are Λ-sensitive in all our tests.

Model: thermal convection + rotation

- Boussinesq 3D **thermal convection** + **global rotation** tilted at latitude ϕ=45° (Hathaway & Somerville 1986; Currie 2014)
- Box domain 5:5:1, periodic in x,y.
- Free-slip, fixed-T boundaries.
- $\mathcal{R}a$ = Rayleigh, convective driving: 4×10⁴ to 2×10⁵
- T_y = meridional thermal gradient: 0, -0.5, -2
- Basic state background:
  - T temperature linear in y and z
  - U zonal velocity linear in z
- **Dedalus** pseudospectral simulations ran at 64×64×64 to 128×128×128 resolution. (Burns+ 2020; https://dedalus-project.org/)

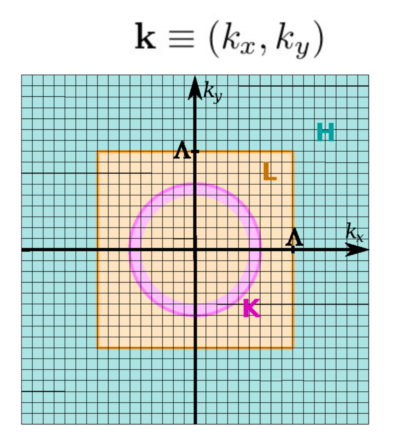
$$\mathcal{R}a = \alpha g T_z L_z^4/\nu\kappa$$

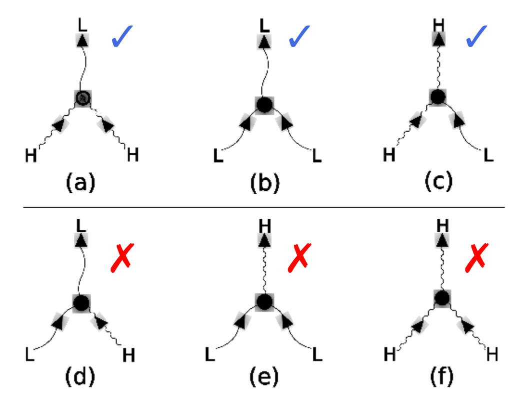
$$T = T_0 + T_z\,z + T_y\,y + T_{\rm L} + T_{\rm H}\ ,$$
$$U = U_z\,(z-\tfrac{1}{2}L_z) + u_{\rm L} + u_{\rm H}\ .$$
$$U_z = -\mathcal{R}a T_y / \mathcal{T}a^{1/2}\sin\phi$$

GQL approximated dynamics

- **Generalised quasilinear** simulations (see Marston+2016; Child+2016; Hernández+2022; Oishi & Baxter 2023)
- Divide the horizontal (k_x,k_y) Fourier space by a square at chosen **cutoff Λ**.
- Decompose all variables q=q_L+q_H into:
  - **low modes** (L) inside square Λ;
  - **high modes** (H) outside square Λ.
- Λ=0 is "**QL**" **quasilinear** e.g. Malkus 1954.
- **Linear terms** $\mathcal{L}$ decompose directly.
- Only some $\mathcal{Q}$ **nonlinear interactions** can remain in each of the split PDEs. Protects relevant conservation laws.

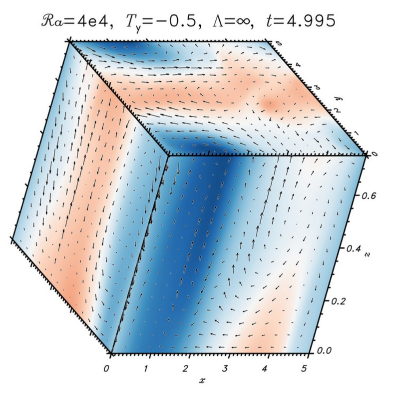
$$\partial_t \mathbf{q}_{\rm L} = \mathcal{L}[\mathbf{q}_{\rm L}] + \mathcal{Q}_{\rm L}[\mathbf{q}_{\rm L},\mathbf{q}_{\rm L}] + \mathcal{Q}_{\rm L}[\mathbf{q}_{\rm H},\mathbf{q}_{\rm H}]\ ,$$
$$\partial_t \mathbf{q}_{\rm H} = \mathcal{L}[\mathbf{q}_{\rm H}] + \mathcal{Q}_{\rm H}[\mathbf{q}_{\rm L},\mathbf{q}_{\rm H}] + \mathcal{Q}_{\rm H}[\mathbf{q}_{\rm H},\mathbf{q}_{\rm L}]\ ,$$





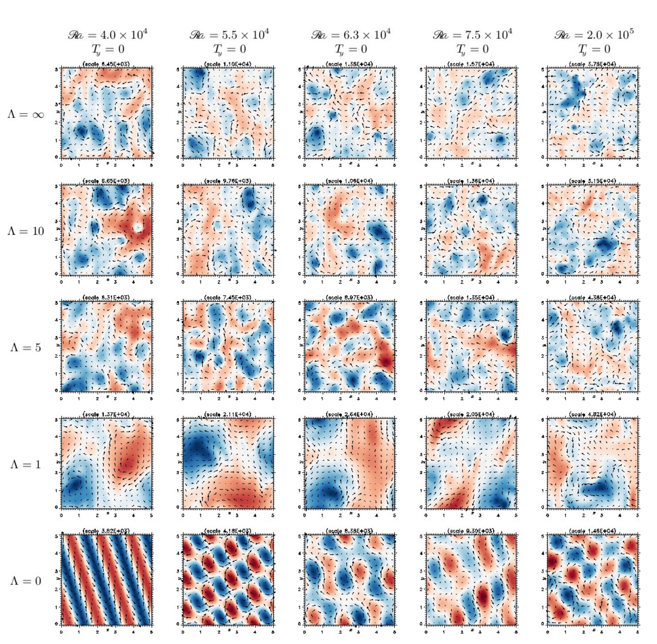
Flow morphology

- **Fully nonlinear** (NL, Λ=∞) reference simulations:
  - T_y = 0: weak thermal wind; convection without preferred horizontal direction;
  - T_y = -0.5: thermal wind, zonal jets; one giant vortex (if Ra is low enough);
  - T_y = -2: strong zonal jets; no permanent vortex. Bursting in some cases.

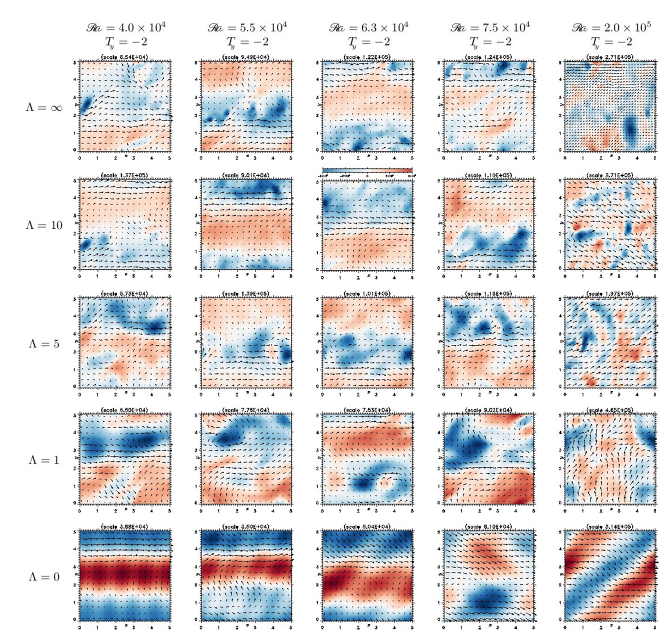


A model where a giant vortex emerges. Red/blue is pressure; arrows are velocities.

- **Quasilinear** (QL, **Λ=0**): unrealistic frozen patterns if T_y=0; zonal banding if T_y=-2.
- **GQL at low Λ**: live flows; overdo **inverse cascades** to **giant vortices** / **zonal flows**.
- **GQL at high Λ≥5**: more accurate flow patterns; better for big $|T_y|$ and **steep profiles**!



Snapshot horizontal sections of pressure and velocity; T_y=0 **weak** thermal wind; at various (Λ,$\mathcal{R}a$)

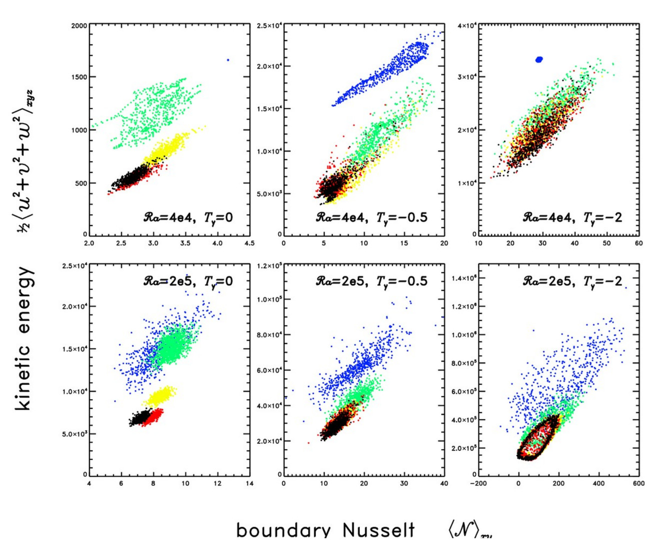


Snapshot horizontal sections of pressure and velocity; T_y=-2 **strong** thermal wind; at various (Λ,$\mathcal{R}a$)

Global properties

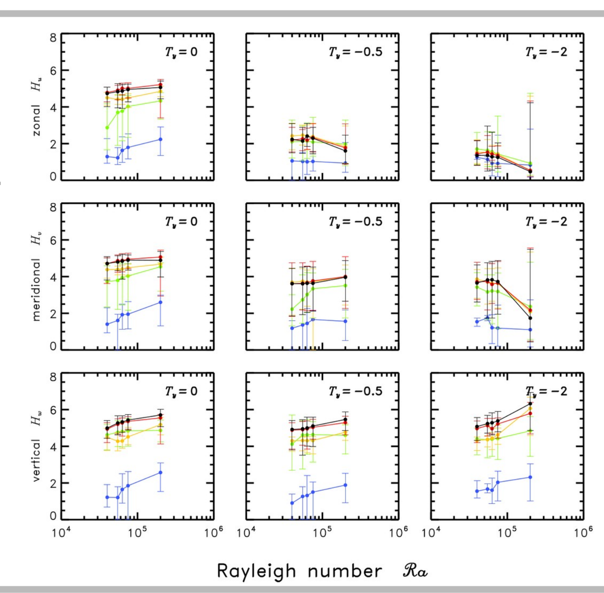
- QL models overestimate **heat transfer** ($\mathcal{N}$) and **kinetic energy** (E).
- Raising the **GQL cutoff** Λ improves the ($\mathcal{N}$,E) distributions.
- Realism improves faster given a **stronger thermal wind** (steeper $|T_y|$; Saxton+2023).

SCATTER-PLOT KEY: Λ=0 (QL), Λ=1 (minimal GQL), Λ=5 (mid GQL), Λ=10 (high GQL), Λ=∞ (nonlinear, NL)

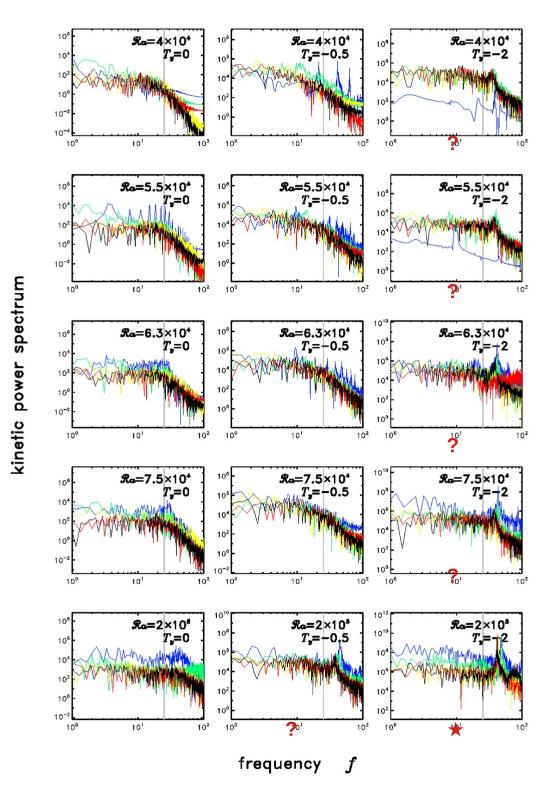


Spatial entropy

- **Spatial entropy** measures disorder in the Fourier power distribution (e.g. Powell & Percival 1979).
- H low for ordered patterns; H high for turbulence.
- Differs among velocity components u, v, w.
- Λ=0 QL and Λ=1 low GQL are too orderly by >3σ.
- High GQL (Λ=5 and 10) are consistent with disorder or self-organisation in NL reality.



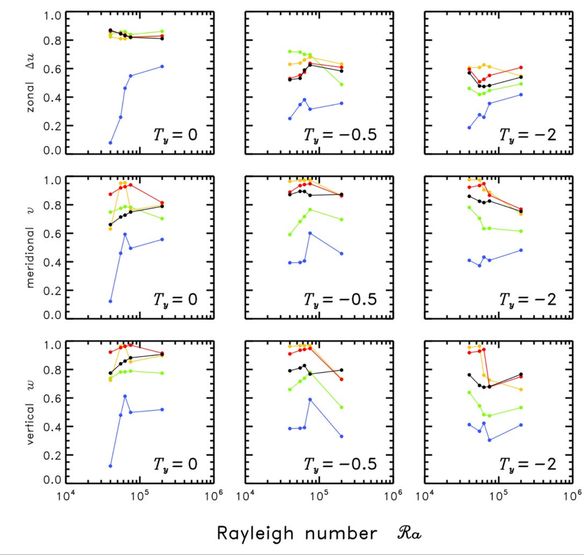
Temporal properties



E.g. power spectra of kinetic energy variability….

- QL (Λ=0) behaves strangely, but this is unsurprising given its locked patterns.
- Break at **rotation frequency** f_* = Ω/2π ≈ 25.
- **White noise** at $f < f_*$; **red noise** at $f > f_*$.
- Six cases have a **QPO** spike?
- Cyclic **bursting** and gyrating wind for the extreme case ($\mathcal{R}a$,T_y)=(2e5,-2) ★
- Time spectra are **mostly unaffected** by GQL.

Spatial anisotropy



- Pick ellipsoidal contours in (k_x,k_y) plane → axis ratio: 0 elongated …. 1 round distribution.
- Some variables are more anisotropic than others.
- Λ=0 QL and Λ=1 low GQL are too anisotropic.
- High GQL (Λ=5 and 10) can clip spectra and **reduce anisotropy**, even in cases where other global statistics are accurate!

Transfer functions

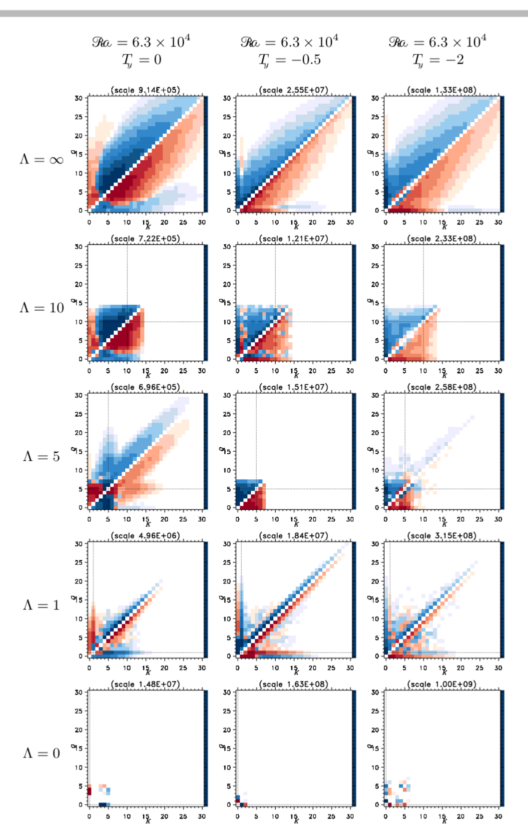
Mask the velocities $\mathbf{u}(x,y,z)$ at wavenumber shells **K** → $\mathbf{u}_K(x,y,z)$ maps (e.g. methods of Alexakis+2005, Favier+2014)

$$\mathsf{K} \equiv \left\{(k_x,k_y) : K < \sqrt{k_x^2+k_y^2} \le K+1\right\}$$

Infer kinetic **power transfer** rate from shells Q to K:

$$\mathcal{T}(Q,K) \equiv -\iiint \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\;\mathbf{u}_K\cdot(\mathbf{u}\cdot\nabla)\mathbf{u}_Q$$

- NL (Λ=∞): direct cascades (+/-) at high k; some inverse cascades (-/+) at low k.
- GQL Λ~5,10 limits the active spectral zone; else OK.
- Minimal GQL (Λ=1): strong inverse cascades at k≈1.
- QL (Λ=0): impaired transfers; few modes active.



Summary

- **GQL** splits **fluctuations** from **background evolution** ⇒ **probe interscale physics**.
- GQL in spatial domain hardly affects **temporal properties.** ✓
- **Minimal GQL** (Λ=1) greatly surpasses the realism of quasilinear models (Λ=0). ✓
- Low Λ over-promotes **inverse cascades** to **giant vortices** and **zonal flows**.
- High Λ≥5 fixes **heat transfer rate** and other global statistics. ✓
- Large $|T_y|$, strong shear profile, enables greater accuracy in GQL. ✓
- Beware that GQL can **isotropise** the system erroneously (even at Λ=10).
- FUTURE WORK: more entropy analyses? magnetic fields?

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